Machine Component Design

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Contents

§1 September 11, 2017

§1.1 Introduction

Machine design deals with the planning, construction, and analysis of machine elements. This course covers the basic principles of machine component design. This will include the design for stiffness, strength, and endurance, Some examples of machine elements include shafts, gears, belts, chains, bearing, and pulleys. There are different types of machine design:

- **Adaptive design** is a design in which we make minor modifications in the existing product. An example is the diesel engine.
- **Developed design** involves the creation of a new design from a previous basis. Examples include electronic devices.
- **New design** involves the creation of a machine component made from scratch. An example could be solar plants.

There are different reasons to design new machine components. The general steps of machine design are summarized below.

1. Need: concerns the decision of whether to purchase a product, or to make a new one.
2. Mechanism: concerns the
3. Analysis of Forces: concerns the types of forces acting on a body or machine element.
4. Material Selection: concerns the appropriate selection of materials in order to reduce costs.
5. Design of Machine Elements: concerns determining the dimensions of the component.
6. Modification of Design: concerns the introduction of small changes to the design.
7. Detailed Drawing: concerns the presentation of complete information regarding drawings, symbols, or specifications.
8. Production or Manufacturing: concerns the actual product, or a 3D printed replica.

§2 September 14, 2017

§2.1 Types of Beams

- A **simply supported beam** has vertical reaction forces at ends A and B.
- A **cantilever beam** is supported by a moment and reaction force at A, and is not supported at B.
- A **fixed beam** is supported by a moment and reaction force at both ends A and B.
- An **overhanging beam** is supported by reaction forces A and B for beam ABC.

Most of class wasted on reviewing shear force and bending moment diagrams.

§3 September 19, 2017

§3.1 Factor of Safety

The *factor of safety* will always be greater than one, and is a ratio of the maximum stress to the working (allowable) stress. For a ductile material, the maximum stress is the *yield stress*. This occurs at the second bump in a stress-strain graph. The *failure (breaking) point* occurs shortly after reaching the yield stress. For the linear region, recall that we can apply *Young's modulus*. For a brittle material, the maximum stress is the *ultimate stress*. For a brittle material, there is no bump, and the breaking point is reach abruptly after reaching maximum stress.

When choosing a factor of safety, we must consider many different factors. The following are some

- Material Selection: Consideration needs to be made with regards to whether the material is ductile or brittle.
- Types of Loading: The factor of safety made be dependent on static, variable, or impact-shock loading.
- Cost: A high factor of safety may result in a higher cost. There must therefore be a balance between cost and factor of safety.
- Importance of Machine Part in Machine: If the machine part plays a critical role, it may require a higher factor of safety. Alternatively, the part may require replacement after a certain number of years.
- Safety to Human Life: Machine failure that may result in the loss of human life should require a higher factor of safety.
- Life of Component: A longer expect lifespan of a component would require a greater factor of safety.

§3.2 Static Body Stresses

When an external load is applied to a member, body stresses exist within a member. There are different types of stress.

- **Tensile stress** is stress that is produced in a member from a pull type of loading, and is denoted by σ_t. It causes elongation along the axis of the applied stress. Tensile stress is given by
$$\sigma = \frac{P}{A}.$$
- **Compressive stress** is stress that is produced in a member due to a push type of loading, and is denoted by σ_c. It causes shortening along the axis of the applied stress. The sign for compressive stress is negative by convention, so the equation for compressive stress is
$$\sigma = -\frac{P}{A}.$$
Both tensile and compressive stress occur under *axial loading*.
- **Shear stress** is stress that is produced in a member when a load is tangential to the cross sectional area, and is denoted by τ. This is given by
$$\tau = \frac{P}{A}.$$

Note that there may be occurrences of double shear, where the area in consideration is doubled. Shear stress occurs under *direct shear loading.*

- **Torsional shear stress** is stress that occurs in a rotating member. It results from the twisting of an object due to an applied torque. This is given by

$$\tau = \frac{Tc}{J},$$

where T is the torque, c is the radius, and J is the polar moment of inertia. For a round bar of radius c, it is given by $J = \pi c^4/2$. If we have a hollow rod with outer radius c_1 and inner radius c_0, then this becomes $J = \pi\left(c_1^4 - c_0^4\right)/2$. For rectangular sections, the maximum shear stress is given by

$$\tau_{max} = \frac{T(2a + 1.8b)}{a^2b^2},$$

where a is the longer length, and b is the shorter length. Torsional stress results from *torsional loading.*

- **Crushing stress** is localized stress acting on a member irrespective of the type of loading. A load P applied is divided by the area of $A = \pi dL$, where d is the diameter of a bolt or pint, and L is the length of the bolt along which the load is exerted.

- **Bearing stress** is stress that is most frequently found in continuously rotating members. The shaft is usually made of a hard material such as steel, while the *bush* is made of a soft material such as rubber. The area is also given by $A = \pi dL$.

- **Bending stress** is stress that results from bending, and is denoted by σ_B. It can be calculated as

$$\sigma_B = \frac{Mc}{I},$$

where M is the bending moment, c is the displacement from the centroid, and I is the second moment of inertia, which is given as $I = \pi c^4/4$ for round bars, and $bh^3/12$ for rectangular sections. Bending stress results from *pure bending loading.* Pure bending occurs when the shear $V = 0$.

- **Transverse shear stress** is stress that is produced as a result of shear along a beam. This is given by

$$\tau = \frac{QV}{It},$$

where V is the shear force, Q is the first moment with respect to the neutral axis, I is the moment of inertia about the same axis, and t is the thickness. In this course, we make use of the generalized formula

$$\tau = \frac{V}{It}\int_{y=y_0}^{y=c} y\,\mathrm{d}A.$$

We may choose to determine the maximum values of shear stress at the neutral axis of solid round sections, thin hollow round sections, and rectangular sections. The expression becomes respectively

$$\tau_{max} = \frac{4V}{3A},$$

$$\tau_{max} = \frac{2V}{A},$$
$$\tau_{max} = \frac{3V}{2A}.$$

Transverse shear stress occurs due to *transverse shear loading*.

§4 September 26, 2017

§4.1 Mohr's Circle Review

Recall that to plot Mohr's circle, we first determine the stresses σ_x, σ_y, and $\tau_{xy} = \tau_{yx}$. We then plot the normal and shear stresses acting on the x and y faces, with τ counterclockwise taken as negative and τ clockwise taken as positive. We then draw a line connecting these two points, indicating the diameter of the circle. Note that angles measured on the circle are twice the corresponding angles on the actual element.

From this, we can determine $\sigma_{avg} = (\sigma_x + \sigma_y)/2$, the radius from $R = \tau_{max} = \sqrt{(\sigma_y - \sigma_{avg})^2 + \tau_{xy}^2}$, the principal stresses $\sigma_{max,min} = \sigma_{avg} \pm \tau_{max}$, and the principal planes from $\tan(2\phi) = \tau_{xy}/(\sigma_y - \sigma_{avg})$.

We can now study three-dimensional stress as it occurs in real three-dimensional bodies. Uniaxial stress (pure tension or compression) involve three principal stresses, with two of them zero. Likewise, biaxial stress has one principal stress zero. A complete Mohr's circle representation in three dimensions would include circles of stresses between σ_1 and σ_2, between σ_1 and σ_3, and between σ_2 and σ_3. The largest of the three Mohr circles represents the maximum shear stress, as well as the two extreme values of normal stress.

Example 4.1

A member at a certain location has three-dimensional stress given by $\sigma_x = 60000$psi, $\sigma_y = 40000$psi, $\sigma_z = -20000$psi, $\tau_{xy} = 10000$psi, $\tau_{yz} = 20000$psi, and $\tau_{zx} = -15000$psi. Determine the principal normal stresses and maximum shear stress.

Solution. For three-dimensional stress, we solve the *characteristic equation* to determine the principal normal stresses. This is given by the equation

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0,$$

where

$$I_1 = \sigma_x + \sigma_y + \sigma_z,$$
$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2,$$
$$I_3 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2.$$

Solving this, we find $\sigma_1 = 69600$psi, $\sigma_2 = 38001$psi, and $\sigma_3 = -27601$psi, where $\sigma_1 > \sigma_2 > \sigma_3$. Now, $\tau_{13} = \|\sigma_1 - \sigma_3\|/2 = 48600$psi, $\tau_{21} = \|\sigma_2 - \sigma_1\|/2 = 15799$psi, and $\tau_{32} = \|\sigma_3 - \sigma_2\|/2 = 3280$psi. Therefore, $\tau_{max} = 49600$psi. ■

On a Mohr diagram, τ_{max} is obtained by plotting the circles for σ_1, σ_2, and σ_3 (last one usually equal 0). Drawing the larger circle that surrounds all of these smaller circles, we find that the radius is τ_{max}.

§5 October 3, 2017

§5.1 Stress Concentration Factors

We note that the uniform distribution of force flow lines is an idealization, and only exists in regions substantially removed from the ends. Near the ends however, the force flow lines indicate concentrations of stress near the outer surface. We can now evaluate the maximum stresses existing in a part.

In recent years, finite-element analysis has resulted in many graphs that give values of the *theoretical stress concentration factor* K_t. This is given as

$$\sigma_{max} = K_t \sigma_{nom},$$

$$\tau_{max} = K_t \tau_{nom},$$

where nom denotes values calculated using the original formulas, and max denotes the maximum corresponding value.

For grooved shafts, we need to consider whether the shaft is under bending, axial load, or torque. Then, comparing the ratio of r with the inner diameter d or the ratio of d to D, we find the corresponding K_t from the corresponding stress concentration factor graphs. These graphs also provide appropriate formulas for calculating the nom values.

Example 5.1

A shaft is supported by bearings at locations A and B and is loaded with a downwards 1000N force at 500mm from A. 250mm to the right of this is B. The diameter at the ends A and B are $40mm$, while the larger diameter in the middle is 80mm. The fillet from end B until the diameter increases is 70mm, and $r = 5$mm. Determine the maximum stress at the shaft fillet.

Solution. First, we need to find the reaction forces by summing the moments at A and B and equating them to zero. Doing so, we find that $A = 333$N, and $B = 667$N. Now, we draw the shear and bending moment diagrams. We can then find the moment at the fillet to be 47Nm. Using this, we calculate $\sigma = Mc/I = 7.5$MPa. We know that the ratio is $r/d = 5/40 = .125$. Doing so, we obtain $K_t = 1.65$, so our final stress is $1.65(7.5) = 12.4$MPa. ■

§5.2 Thermal Stresses

Recall that stresses caused by constrained expansion and contraction may be due to temperature changes or material phase changes. When the temperature of an unrestrained homogeneous body is changed uniformly, then

$$\epsilon = \alpha \Delta T,$$

where ϵ is the strain, α is the coefficient of thermal expansion, and ΔT is the change in temperature. Unrestrained volume change produces no shear strain and no stresses. However, if restraints are placed on the body undergoing temperature change, then the stresses can be determined by the *superposition principle* by first considering the dimension changes resulting from the temperature change, then considering the required loads required to enforce the restrained dimensions. The resulting stresses can then be computed from these loads.

Example 5.2

A 250mm length of steel tubing (with properties of $E = 207 \cdot 10^9$Pa and $a = 12 \cdot 10^{-6}/°$C) having a cross-sectional area of 625mm^2 is installed with fixed ends so that it is stress-free at $26°C$. In operation, the tube is heated throughout to a uniform $249°C$. Careful measurements indicate that the fixed ends separate by 0.20mm. Determine the loads that are exerted on the ends of the tube, and what the resultant stresses are.

Solution. For an unrestrained tube, we have $\epsilon = \alpha\Delta T = 12*10^(-6)*(249-26) = 0.002676$. Thus, $\Delta L = L\epsilon = 250(0.002676) = 0.6675$mm. Since the measured expansion was only 0.2mm, the constraints must apply forces sufficient to produce a deflection of $\delta = 0.6675 - 0.2 = 0.4675$mm. Using the equation $\delta = PL/AE$, we get $\sigma = P/A = 387.09$MPa. ■

§6 October 5, 2017

§6.1 Failure Theories

Consider a test of the yield strength of a material under tension. The theory behind the various classical failure theories is that whatever is responsible for failure in the standard tensile test will also be responsible for failure under all other conditions for static loading. For instance, suppose that the theory states that failure occurs during the tensile test because the material was unable to withstand a certain tensile stress. The theory then predicts that under any conditions of loading, the material will fail if and only if normal stress exceeds that value. On the other hand, suppose that a theory claims that failure during the tensile test occurs because the material is limited in its ability to resist a certain shear stress. Failure would then occur when this value is exceed in shear.

Example 6.1

Under the general loading of a proposed application, a certain material has $\sigma_1 = 80$, $\sigma_2 = -40$, and $\sigma_3 = 0$. By plotting on Mohr's circle, we find that $\tau_{max} = 60$. Under the standard tensile test of that same material where only tension is applied, we have $\sigma'_1 = 100$ and $\sigma'_2 = \sigma'_3 = 0$. Drawn on Mohr's circle, this gives $\tau max' = 50$. If the theory was based on tensile stress, then failure would not be expected in the proposed application, since $80 < 100$. However, if the theory was based on shear, then failure would be expected to occur in the proposed application since $60 > 50$.

Various failure theories are presented below.

- **Maximum Normal Stress Theory**: Failure will occur whenever the greatest tensile or compressive stress is greater than the uniaxial tensile strength or the uniaxial compressive strength respectively. This is very suitable for brittle fracture, but is not suitable to predict failure for ductile materials. Consider two Mohr circles corresponding to uniaxial compressive strength S_{uc} to the left of the origin and to uniaxial tensile strength S_{ut} to the right of the origin. So long as the normal stress σ is between these two values, the material will not fail.

- **Maximum Shear Stress Theory**: A material subject to any combination of loads will fail by yielding or fracturing whenever the maximum shear stress exceeds the shear strength (yield or ultimate) of the material. Thus, we plot S_{yt} and obtain τ_{max} from the plot of of the corresponding Mohr circle. If τ_{max} is exceeded in a particular application, we would expect failure to occur.

- **Maximum Distortion Energy Theory** Any elastically stressed material undergoes a slight change in shape or volume. The energy required to produce this change is stored in the material as elastic energy. The formula to obtain the equivalent elastic stress is

$$\sigma_e = \frac{\sqrt{2}}{2}\sqrt{(\sigma_2-\sigma_1)^2+(\sigma_3-\sigma_1)^2+(\sigma_3-\sigma_2)^2}.$$

In the case for biaxial stress and direct stress, we obtain the following equations respectively.

$$\sigma_e = \sqrt{\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2},$$

$$\sigma_e = \sqrt{\sigma_x^2+\sigma_y^2-\sigma_x\sigma_y+3\tau_{xy}^2}.$$

Note that if only σ_x and τ_{xy} exist, then the last formula simplifies to

$$\sigma_e = \sqrt{\sigma_x^2+3\tau_{xy}^2}.$$

Once the equivalent stress is obtained, this is compared with the yield strength from the standard tensile test. If σ_e exceeds S_{yt}, yielding is predicted.

Example 6.2

Strain gage tests have established that the critical location on the surface of a steel part is subjected to principal stresses of $\sigma_1 = 35\text{ksi}$ and $\sigma_2 = -25\text{ksi}$. The surface is exposed and unloaded, so $\sigma_3 = 0$. The steel has a yield strength of $S_{yt} = 100\text{ksi}$. Estimate the safety factor with respect to initial yielding using the preferred theory. Compare this with results given by other failure theories.

Solution. For distortion energy theory, we obtain

$$\begin{aligned}\sigma_e &= \sqrt{\sigma_1^2+\sigma_2^2\sigma_1\sigma_2} \\ &= \sqrt{35^2+(-25)^2-(35)(-25)} \\ &= 52.2\end{aligned}$$

The factor of safety is therefore $SF = S_{yt}/\sigma_e = 100/52.5 = 1.9$. For shear stress theory, we find that the principal stresses define a Mohr circle with a radius of $\|\sigma_1-\sigma_2\|/2 = (35-(-25))/2 = 30$. The standard tensile test gave a principal Mohr circle with a radius of $S_{yt}/2 = 100/2 = 50$. Thus, the safety factor is $SF = 50/30 = 1.7$. Using normal stress theory, we have a maximum normal stress of 35ksi. Thus, the factor of safety is $SF = 100/35 = 2.9$. ■

§7 October 12, 2017

§7.1 Fatigue

The concept of *fatigue* concerns the small (and often microscopic) cracks at critical areas of high local stress. Fatigue failure occurs as a result of repeated plastic deformations, such as the breaking of a wire after bending back and forth repeatedly. Fatigue failure often occurs after many thousands of cycles of minute yielding at the microscopic level. Thus, this type of failure can occur at stress levels far below the traditional yield point or elastic limit. Strengthening vulnerable locations such as holes and sharp corners is often as effective as making the part out of a stronger material. The R. R. Moore fatigue test is often used to determine the fatigue strength characteristics of materials.

Tests against various weights allow one to produce *S-N curves*. The intensity of reversed stress causing failure after a given number of cycles is the *fatigue strength* corresponding to that number of loading cycles. Numerous tests have established that ferrous materials have an *endurance limit*, defined as the highest level of alternating stress that can be withstood indefinitely without failure. For materials where the endurance limit is clearly defined, the *knee* denotes the first point at which the slope on the graph becomes zero. The usual symbol for endurance limit is S_n, while S_n' is used to denote standard lab conditions. If we do not have lab data for S_n' or S', we can approximate the endurance limit with correction factors

$$S_n = S_n' C_L C_G C_S C_T C_R,$$

where S_n' is obtained from the R. R. Moore endurance test, C_L is the load factor, C_G is the size (gradient) factor, C_T is the temperature factor, C_S is the surface factor, and C_R is the reliability factor. S_u is the ultimate tensile strength, and S_{us} is the ultimate shear stress.

1. The value of the endurance limit under lab conditions can be obtained through experiment or through approximation. For steel, we have

$$Sn' = \begin{cases} 0.5S_u & \text{if } S_u \leq 1400\text{MPa or } 200\text{ksi}, \\ 100\text{ksi or } 700\text{MPa} & \text{if } S_u > 1400\text{MPa or } 200\text{ksi}. \end{cases}$$

   For iron, we have

$$Sn' = \begin{cases} 0.4S_u & \text{if } S_u \leq 60\text{ksi}, \\ 24\text{ksi} & \text{if } S_u > 60\text{ksi}. \end{cases}$$

2. The **surface factor** C_S relates the amount of surface damage that can be caused by commercial processes based on the susceptibility of the material to damage. The surface factor changes for different finishes applied to steels of various hardnesses. It is obtained from Figure 8.13 of the textbook, or from

$$C = e(S_u)^f,$$

   where the constants e and f are obtained from the following table.

Manufacturing Process	Factor e (MPa)	Factor e (ksi)	Exponent f
Grinding	1.58	1.34	-0.085
Machining or Cold Drawing	4.51	2.70	-0.265
Hot Rolling	57.7	14.4	-0.718
As Forged	272.0	39.9	-0.995

Example 7.1

Let $S_u = 520$MPa, and the surface be a grinding surface. From the table, we find that $e = 1.58$MPa and $f = -0.085$. Therefore, $C_S = 1.58(520)^{-0.085} \approx 0.9285$.

3. The **size (gradient) factor** C_G accounts for the differences in sizes of specimens being tested. It is found using Table 8.1. If d is different from 0.3in, and it is subjected to reversed bending or torsion, then it should carry a size factor of $C_G = 0.9$. The same is true for axial loads. We can also use the following formulas for determining C_G where values are in inches

$$C_G = \begin{cases} \left(\frac{d}{0.3}\right)^{-0.107} = 0.879d^{-0.107} & 0.11 \leq d \leq 2, \\ 0.91d^{-0.157} & 2 \leq d \leq 10. \end{cases}$$

   Alternatively, we can use the following formula for values in millimeters.

$$C_G = \begin{cases} \left(\frac{d}{7.62}\right)^{-0.107} = 1.24d^{-0.107} & 2.79 \leq d \leq 51, \\ 1.51d^{-0.157} & 51 \leq d \leq 254. \end{cases}$$

   If instead the loading is axial, then $C_G = 1$. If the stationary shaft is under a completely reverse torsional load, then we calculate by using the previous equation. A circular or rectangular section that is not rotating is subject to reverse bending.

4. The **loading factor** C_L relates the fact that it may not have the same loading as in lab conditions. If it is in bending (as in the lab) or in axial or combined loading, then $C_L = 1$. For pure torsional loading, $C_L = 0.58$.

5. The **temperature factor** C_T accounts for the fact that the strength of a material decreases with increased temperature. It can be obtained from Table 8.1. Alternatively, it can be obtained from

$$C_T = 0.875 + 0.432\left(10^{-3}\right)T_f,$$

   where T_f is the temperature in Fahrenheit.

6. The **reliability factor** C_R relates that a more reliable estimate of the endurance limit requires using a lower value of the endurance limit. It is obtained from Table 8.1. For higher reliability, we desire to be more conservative.

Example 7.2

Estimate the $S - N$ curve for the axial loading of precision commercially polished steel parts having $S_u = 150$ksi and $S_y = 120$ksi. The cross sectional diameter is less than 2in. Determine the peak alternating stress S at 10^4 cycles.

Solution. We assume that $C_R = C_T = 1$ and take the gradient factor to be $C_G = 0.9$ from Table 8.1. For the 10^3 cycle, we note that $S_f = 0.75S_uC_T = 0.75(150)(1) = 112.5$ksi. For the 10^6 cycle, we approximate $S_n' = 0.5S_u = 0.5(150) = 75$ksi since we lack additional data. Additionally, $C_L = 1$, and Figure 8.13 for polished steel at $S_u = 150$ksi is $C_S = 0.9$. Thus, we find that $S_n = S_n'C_LC_GC_SC_TC_R = 75(1)(0.9)(0.9)(1)(1) = 60.75$ksi. We can then plot 112.5ksi at 10^3 and 60.75ksi at 10^6. To determine the peak alternating stress S at a particular number of cycles in this range, we solve for S in

$$\frac{\log(112.5) - \log(60.75)}{6-3} = \frac{\log(S) - \log(60.75)}{6 - \log\left(10^4\right)}.$$

Doing so, we find that $S = 91.6$ksi. ∎

§8 October 19, 2017

§8.1 Machine Components

Machine components can be classified into those that **support mechanically generated loads** (such as welded joints, fasteners, and non-permanent joints), and those that **transform mechanical power** through storage (springs and flywheels), dissipation (brakes and clutch), and transmission. There are different types of transmissions.

- Rotational to Linear Motion: An example would be power screws. Power screws are used to convert rotational motion to linear motion. This includes vices, testing machines, presses, lathes, universal machines, and milling machines.
- Fluid to Rotational or Linear: Hydraulics and pneumatics are common examples.
- Rotational to Rotational: Gears, pulleys, belts, chains, bearings, shafts, seals, key couplings, and splines are examples.

§8.2 Power Screws

The following are common types of threads that are used for power screws.

1. **Square Threads**: The pitch p is from when the pattern repeats itself again horizontally. The height is h, and the width is $w = p/2$. This maximizes the efficiency and reduces the radial power. It is difficult to cut with taps and dies. It is a single point tool. Examples include screw jacks, presses, and clamping devices.
2. **ACME Threads**: The pitch p is again from the distance that separates a repeated instance of the pattern. The width is $w = .37p$, and the height is $h = 0.5p + 0.25$. the angle reduces the efficiency compared to square threads. This is more easy to manufacture compared to square threads.
3. **Buttress Threads**: The pitch is the distance at which the pattern repeats. The width is $w = 0.125p$ and the height is $h = 0.75p$.

We shall derive an expression for the effort P applied at the circumference of the screw to lift the load. Because the load is being lifted, the force of friction is $F = \mu R_N$. Resolving forces along the plane and perpendicular to the plane, we obtain respectively,

$$P\cos(\alpha) = W\sin(\alpha) + F = W\sin(\alpha + \mu R_N,$$

$$R_N = P\sin(\alpha) + W\cos(\alpha).$$

Substituting the expression for R_N into the first equation above, we obtain the following.

$$\begin{aligned} P\cos(\alpha) &= W\sin(\alpha) + \mu\left(P\sin(\alpha) + W\cos(\alpha)\right) \\ P(\cos(\alpha) - \mu\sin(\alpha)) &= W(\sin(\alpha) + \mu\cos(\alpha)) \\ P &= \frac{W(\sin(\alpha) + \mu\cos(\alpha))}{\cos(\alpha) - \mu\sin(\alpha)} \end{aligned}$$

Now, we can substitute $\mu = \tan(\phi)$ into the equation.

$$\begin{aligned} P &= \frac{W\left(\sin(\alpha) + \tan(\phi)cos(\alpha)\right)}{\cos(\alpha) - \tan(\phi)\sin(\alpha)} \\ &= \frac{W\left(\sin(\alpha)\cos(\phi) + \sin(\phi)cos(\alpha)\right)}{\cos(\alpha)\cos(\phi) - \sin(\phi)\sin(\alpha)} \\ &= \frac{W\sin(\alpha+\phi)}{\cos(\alpha+\phi)} \\ &= W\tan(\alpha+\phi) \end{aligned}$$

§9 October 24, 2017

§9.1 Torque Required to Raise Load by Square Threaded Screws

1. Let L be the length of the horizontal lever, p be the pitch of the screw, d be the mean diameter of the screw, α be the helix angle, P be the effort applied at the circumference of the screw to lift the load, W be the load to be lifted, and $\mu = \tan(\phi)$ be the coefficient of friction between the screw and the nut where ϕ is the friction angle.

2. The mean diameter can be obtained from
$$d = \frac{d_o + d_c}{2} = d_o - \frac{p}{2} = d_c + \frac{p}{2},$$
where d_c is the core (inner, root, or minor) diameter and d_o is the nominal (outside or major) diameter.

3. From the geometry, we find that $\tan(\alpha) = p/\pi d$, where α is the helix angle.

4. The ideal effort neglecting friction is $P_o = W\tan(\alpha)$, but the actual effort applied to rotate the screw is
$$P = W\tan(\alpha+\phi),$$
where ϕ is obtained by considering friction.

5. Efficiency of screw thread is P_o/P, so it is
$$\eta = \frac{W\tan(\alpha)}{W\tan(\alpha+\phi)} = \frac{\tan(\alpha)}{\tan(\alpha+\phi)}.$$

6. The torque required to rotate the screw is
$$T_1 = \frac{Pd}{2} = \frac{W\tan(\alpha+\phi)d}{2}.$$

7. When the axial load is taken up by a thrust collar, the load does not rotate with the screw. Thus, we can determine the torque required to overcome collar friction.
$$T_2 = \mu_1 W\left(\frac{R_1+R_2}{2}\right) = \mu_1 WR,$$
where R_1 is the outside radius of the collar, R_2 is the inside radius of the collar, $R = (R_1+R_2)/2$ is the mean radius of the collar, and μ_1 is the coefficient of friction for the collar. The equation above is used assuming uniform wear conditions.

Alternatively, is we were to assume uniform pressure conditions, we obtain the following formula instead.

$$T_2 = \frac{2}{3}\mu_1 W \left(\frac{(R_1)^3 - (R_2)^3}{(R_1)^2 - (R_2)^2}\right).$$

8. The total torque required to overcome friction and rotate the screw can be found.

$$T = T_1 + T_2.$$

Remark 9.1. The speed of the screw in revolutions per minute is the speed in mm/min divided by pitch of screw in mm. For acme threads, $\mu_1 = \mu/cos(B)$, where the angle from the vertical to one slanted side is B.

§9.2 Torque Required to Lower Load by Square Threaded Screws

The friction force will be acting upwards. Therefore, the resolving force along the slanted plane gives $P\cos(\alpha) = F - W\sin(\alpha)$. Resolving forces perpendicular to this plane s $N = W\cos(\alpha) - P\sin(\alpha)$. Substituting the second equation into the first, we obtain

$$\begin{aligned}
P\cos(\alpha) &= \mu(W\cos(\alpha) - P\sin(\alpha)) - W\sin(\alpha)\\
P(\cos(\alpha) + \mu\sin(\alpha)) &= W(\mu\cos(\alpha) - \sin(\alpha))\\
P &= W\frac{\mu\cos(\alpha) - \sin(\alpha)}{\cos(\alpha) + \mu\sin(\alpha)}\\
&= W\frac{\tan(\phi)\cos(\alpha) - \sin(\alpha)}{\cos(\alpha) + \tan(\phi)\sin(\alpha)}\\
&= W\frac{\sin(\phi)\cos(\alpha) - \sin(\alpha)\cos(\phi)}{\cos(\alpha)\cos(\phi) + \sin(\phi)\sin(\alpha)}\\
&= W\frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)}\\
&= W\tan(\phi - \alpha)
\end{aligned}$$

Remark 9.2. Contrast this with raising the load, which requires $P = W\tan(\phi + \alpha)$. Since $T = Pd/2$, this becomes $T = W\tan(\phi - \alpha)d/2$.

Example 9.3

A vertical screw with single start square threads of 50mm mean diameter and 12.5mm pitch is raised against a load of 10 kN by means of a hand wheel, the boss of which is threaded to act as a nut. The axial load is taken up by a thrust collar which supports the wheel boss and has a mean diameter of 60mm. The coefficient of friction is 0.15 for the screw and 0.18 for the collar. If the tangential force applied by each hand to the wheel is 100N, find suitable diameter of the hand wheel.

Solution. Let $d = 50$mm, $p = 12.5$mm, $W = 10$kN, $R = 30$mm, $\mu = \tan(\phi) = 0.15$, $\mu_1 = 0.18$, and $P_1 = 100$N. We want to find the diameter D_1. We know that $\tan(\alpha) = p/\pi d = 12.5/50\pi = 0.08$. Now, we know that $P = W\tan(\alpha + \phi) = W(\tan(\phi) + \tan(\alpha))/(1 - \tan(\phi)\tan(\alpha)) = 10000(0.08 + 0.15)/(1 - 0.08 * 0.15) = 2328$N. But then the torque required to turn the hand wheel is $T = Pd/2 + \mu_1 WR = 2328(50)/2 + 0.18(10000)(30) = 112200$Nmm. We know the torque applied to the hand wheel is $T = 2P_1(D_1)/2 = 2(100)(D_1)/2$, we equate both expressions for torque to find that $D_1 = 1122$mm. ■

§10 October 31, 2017

§10.1 Design of Screw Jack

There are various parts of a screw jack. To design a screw jack, we need to consider the screwed spindle with square threaded screws, the nut and collar for the nut, the head at the top of the screwed spindle for the handle, the cup at the top of the head for the load, and the body of the screw jack. To design a screw jack for a load W, we adopt the following procedure.

1. We first need to find the core diameter d_c by considering the screw under pure compression.
$$W = \sigma_c A_c = \frac{\sigma_c \pi (d_c)^2}{4},$$
where σ_c is the compressive stress.

2. Find the torque T_1 required to rotate the screw and find the shear stress τ due to this torque.
$$T_1 = \frac{Pd}{2} = \frac{W \tan(\alpha + \phi) d}{2},$$
where P is the load effort required at the circumference of the screw and d is the mean diameter of the screw.

3. The shear stress due to torque T_1 with core diameter d_c can then be found.
$$\tau = \frac{16 T_1}{\pi (d_c)^3}.$$

4. The direct compressive stress due to an axial load can also be found. The first equation above can be rearranged to find the direct compressive stress σ_c due to the axial load.
$$\sigma_c = \frac{4W}{\pi (d_c)^2}.$$

5. Next, the principal stresses are found. The following expressions are used to find the maximum principal stress (tensile or compressive) and the maximum shear stress.
$$\sigma_{c(max)} = \frac{1}{2}\left(\sigma_c + \sqrt{(\sigma_c)^2 + 4\tau^2}\right),$$
$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2},$$
where both of these stresses should be less than the permissible stresses.

6. Find the height h of the nut considering the bearing pressure on the nut.
$$P_b = \frac{W}{\frac{\pi}{4}\left((d_0)^2 - (d_c)^2\right) n},$$
where n is the number of threads in contact with the screw spindle and P_b is the bearing pressure. Solving for n, this can be used to find the height of the nut $h = np$, where p is the pitch of the threads.

7. Check the stresses in the screw and the nut.

$$\tau_{screw} = \frac{W}{\pi n d_c t},$$

$$\tau_{nut} = \frac{W}{\pi n d_0 t},$$

where t is the thickness of the screw given by $t = p/2$.

8. Find the inner diameter D_1 and the outer diameter D_2. The inner diameter D_1 is obtained by considering the tearing strength of the nut, while the outer diameter D_2 is found by considering the crushing strength of the nut collar.

$$W = \frac{\pi}{4}\left((D_1)^2 - (d_0)^2\right)\sigma_t,$$

$$W = \frac{\pi}{4}\left((D_2)^2 - (D_1)^2\right)\sigma_c.$$

9. The thickness t_1 of the nut collar is found by considering the shearing strength of the nut collar.

$$W = \pi D_1 t_1 \tau.$$

10. Next, we fix the dimensions for the diameter of head D_3 on the top of the screw and for the cup. Take $D_3 = 1.75 d_0$. The seat for the cup is made equal tot eh diameter of head and it is chamfered at the top. The cup is fitted with a pin of diameter of approximately $D_4 = D_3/4$. This pin remains a loose fit in the cup.

11. Find the torque T_2 required to overcome friction at the top of the screw. Assuming uniform pressure conditions and alternatively assuming uniform wear conditions, we obtain the following equations respectively.

$$T_2 = \frac{2}{3}\mu_1 W \left(\frac{(R_3)^3 - (R_4)^3}{(R_3)^2 - (R_4)^2}\right),$$

$$T_2 = \mu_1 W \frac{R_3 + R_4}{2} = \mu_1 W R,$$

where $R_3 = D_3/2$ is the radius of head, and $R_4 = D_4/2$ is the radius of pin.

12. Now, the total torque to which the handle will be subjected can be determined.

$$T = T_1 + T_2.$$

Assuming that a person can apply a force of around $300 - 400$N, the length of the handle required is $T/300$.

13. The diameter of the handle D may be obtained by considering the bending effects.

$$M = \frac{\pi}{32}\sigma_b D^3,$$

where σ_b is equal to σ_t or σ_c.

14. The height of the head H is usually taken to be twice the diameter of the handle, so $H = 2D$.

15. We now check the screw for buckling load. The effective length (or unsupported length) of the screw can be determined.

$$L = \text{lift of screw} + \frac{1}{2}\text{ height of nut.}$$

The buckling or critical load can the be calculated.

$$W_{cr} = A_c\sigma_y\left(1 - \frac{\sigma_y}{4C\pi^2 E}\left(\frac{L}{k}\right)^2\right),$$

where σ_y is the yield stress, C is the end fixity coefficient, and $k = d_c/4$ is the radius of gyration. The screw is considered to be a strut with the lower end fixed and the load end free. For one end fixed and the other free, this means that $C = 0.25$. The buckling load as obtained above must be greater than the load at which the screw is designed.

16. Fix the dimensions for the body of the screw jack.

17. Find the efficiency of the screw jack.

§11 November 2, 2017

§11.1 Screw Jack Example

Example 11.1

A screw jack is to lift a load of 80kN through a height of 400mm. The elastic strength of the screw material in tension and compression is 200MPa and in shear 120MPa. The material for the nut is phosphor-bronze for which the elastic limit may be taken as 100MPa in tension, 90MPa in compression, and 80MPa in shear. The bearing pressure between the nut and the screw is not to exceed 18N/mm^2. Design and draw the screw jack. The design should include the design of the screw, the nut, the handle and cup, and the body.

Solution. We know that $W = 80\text{kN}$ at $\sigma_{et} = \sigma_{ec} = 200\text{MPa}$.

$$W = \sigma_c A_c = \frac{\pi}{4}(d_c)^2\frac{\sigma_{ec}}{FS} = \frac{\pi}{4}(d_c)^2\frac{200}{FS}.$$

Using a factor of safety of 2, we find that $d_c = 32\text{mm}$. For square threads of normal series, we select the dimensions of the screw from Table 17.2. Doing so, we obtain the outside diameter of the spindle $d_0 = 46\text{mm}$ and the pitch of threads $p = 8\text{mm}$. From this table, we see that the next highest value of 32mm for the core diameter is 33mm. However, by taking the core diameter to be this value, this results in higher principal stresses than the permissible values. Thus, the core diameter is chosen to be 38mm. We can now determine the mean diameter of the screw and find that

$$d = \frac{d_0 + d_c}{2} = \frac{46 + 38}{2} = 42\text{mm},$$

$$\tan(\alpha) = \frac{p}{\pi d} = \frac{8}{42\pi} = 0.0606.$$

Assuming that the coefficient of friction between the screw and the nut to be $\mu = \tan(\phi) = 0.14$, the torque to rotate the screw in the nut becomes

$$T_1 = \frac{W\tan(\alpha+\phi)d}{2} = W\left(\frac{\tan(\alpha)+\tan(\phi)}{1-\tan(\alpha)\tan(\phi)}\right)\frac{d}{2} = 80000\left(\frac{0.0606+0.14}{1-(0.0606)(0.14)}\right)\frac{42}{2}.$$

Solving this, we obtain $T_1 = 340000$Nmm. Now, the compressive stress due to the axial load and the shear stress due to the torque can be calculated.

$$\sigma_c = \frac{W}{A_c} = \frac{W}{\frac{\pi}{4}(d_c)^2} = \frac{80000}{\frac{\pi}{4}(38)^2} = 70.53\text{N/mm}^2,$$

$$\tau = \frac{16T_1}{\pi(d_c)^3} = \frac{16(340000)}{\pi(38)^3} = 31.55\text{N/mm}^2.$$

Thus, the maximum principal and shear stresses are

$$\sigma_{c(max)} = \frac{1}{2}\left(\sigma_c + \sqrt{(\sigma_c)^2+4\tau^2}\right) = \frac{1}{2}\left(70.53+\sqrt{70.53^2+4(31.55)^2}\right) = 82.58\text{N/mm}^2,$$

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2+4\tau^2} = \frac{1}{2}\sqrt{70.53^2+4(31.55)^2} = 47.315\text{N/mm}^2.$$

But the given value of $\sigma_c = \sigma_{ec}/FS = 200/2 = 100\text{N/mm}^2$ and the given value of $\tau = \tau_e/FS = 120/2 = 60\text{N/mm}^2$. Since the maximum stresses are within limits, the design of the screw for spindle is safe.

Now designing for the nut, let n be the number of threads in contact with the screwed spindle, $h = np$ be the height of the nut, and $t = p/2 = 8/2 = 4$mm be the thickness of the screw, where p is the pitch of the threads found previously. Assuming that the load is distributed uniformly over the cross-sectional area of the nut, with the knowledge that the bearing pressure is $P_b = 18\text{N/mm}^2$,

$$18 = \frac{W}{\frac{\pi}{4}\left((d_0)^2-(d_c)^2\right)n} = \frac{80000}{\frac{\pi}{4}\left(46^2-38^2\right)n} = \frac{151.6}{n}.$$

Solving, we find that $n = 8.4$, which we round to 10 threads. Thus, $h = np = 10(8) = 80$mm. Since $t = p/2 = 8/2 = 4$mm, we can check the stresses induced in the screw and nut.

$$\tau_{screw} = \frac{W}{\pi n d_c t} = \frac{80000}{\pi(10)(38)(4)} = 16.15\text{N/mm}^2,$$

$$\tau_{nut} = \frac{W}{\pi n d_0 t} = \frac{80000}{\pi(10)(46)(4)} = 13.84\text{N/mm}^2.$$

Since these stresses are also within the permissible limit, the design for the nut is safe. We now find the inner diameter, the outer diameter, and the thickness t_1 of the nut collar using $\sigma_t = \sigma_{et}/FS = 100/2$MPa, $\sigma_c = \sigma_{ec}/FS = 90/2$MPa, and $\tau = \tau_e/FS = 80/2$MPa.

$$80000 = \frac{\pi}{4}\left((D_1)^2-(d_0)^2\right)\sigma_t = \frac{\pi}{4}\left((D_1)^2-(46)^2\right)\frac{100}{2} = 39.3\left((D_1)^2-2116\right).$$

Solving this gives $D_1 = 65$mm. This is used to find the outer diameter.

$$80000 = \frac{\pi}{4}\left((D_2)^2-(D_1)^2\right)\sigma_c = \frac{\pi}{4}\left((D_2)^2-(65)^2\right)\frac{90}{2} = 35.3\left((D_2)^2-4225\right).$$

Solving this gives $D_2 = 80.6$. We will say that this rounds to 82mm. Lastly, the thickness of the nut collar can be found.

$$80000 = \pi D_1 t_1 \tau = \pi(65)t_1\frac{80}{2} = 8170t_1.$$

Solving this gives $t_1 = 9.8$, or approximately 10mm.

Next, we design for the handle and the cup. The diameter of the head is $D_3 = 1.75d_0 = 1.75(46) = 80.5 \approx 82$mm. Thus, $D_4 = 82/4 = 20.5 \approx 20$mm. Now, assuming uniform pressure conditions, we can find the torque T_2 required to overcome friction at the top of the screw assuming that $\mu_1 = \mu = 0.14$.

$$T_2 = \frac{2}{3}\mu_1 W \left(\frac{(R_3)^3 - (R_4)^3}{(R_3)^2 - (R_4)^2}\right) = \frac{2}{3}(0.14)(80000)\left(\frac{\left(\frac{82}{2}\right)^3 - \left(\frac{20}{2}\right)^3}{\left(\frac{82}{2}\right)^2 - \left(\frac{20}{2}\right)^2}\right) = 7470\left(\frac{41^3 - 10^3}{41^2 - 10^2}\right).$$

Thus, we find that $T_2 = 321000$Nmm, and then apply this to find the total torque.

$$T = T_1 + T_2 = 340000 + 321000 = 661000\text{Nmm}.$$

Assuming that a force of 300N is applied by the person, the length of the handle is $661000/300 = 2203$mm. Allowing some length for gripping, we take the length of the handle to be 2250mm. Considering that an excessive force applied to the end of the lever causes bending, we find that the maximum bending moment of the handle is $M = 300(2250) = 675000$Nmm. Assuming that the material handle is the same as that of the screw, we have $\sigma_b = \sigma_t = \sigma_{et}/2 = 200/2 = 100\text{N/mm}^2$.

$$675000 = \frac{\pi}{32}(100)D^3 = 9.82D^3.$$

Solving, we find that $D = 40.96 \approx 42$mm. Thus, the height of the head is $H = 2D = 2(42) = 84$mm. Now, the effective length of the buckling screw can be determined. From the problem description, we know that we need to lift through 400mm, and from previous calculation, we found that $h = 80$mm.

$$L = \text{lift of screw} + \frac{1}{2}\text{ height of nut} = 400 + \frac{80}{2} = 440\text{mm}.$$

By taking $\sigma_y = \sigma_{et} = 200$MPa, $C = 0.25$, and $k = d_c/4 = 38/4 = 9.5$mm, we can calculate the critical load.

$$W_{cr} = \frac{\pi}{4}(38)^2(200)\left(1 - \frac{200}{4(0.25)\pi^2 210000}\left(\frac{440}{9.5}\right)^2\right) = 226852(1 - 0.207) = 179894\text{N}.$$

Since the critical load is more than the load that the screw is designed for at 80000N, there is no chance for the screw to buckle.

Various dimensions of the body can be fixed as follows.

$$\begin{gathered}
D_5 = 1.5D_2 = 1.5(82) = 123\text{mm}, \\
t_3 = 0.25d_0 = 0.25(46) = 11.6 \approx 12\text{mm}, \\
D_6 = 2.25D_2 = 2.25(82) = 185\text{mm}, \\
D_7 = 1.75D_6 = 1.75(185) = 320\text{mm}, \\
t_2 = 2t_1 = 2(10) = 20\text{mm}, \\
h_{body} = \text{lift of screw} + \text{ height of nut} + 100 = 400 + 80 + 100 = 580\text{mm}.
\end{gathered}$$

We can additionally find the efficiency of the screw jack. Neglecting friction, the torque T_0 required to rotate the screw can be determined.

$$T_0 = \frac{W\tan(\alpha)d}{2} = 80000(0.0606)\left(\frac{42}{2}\right) = 101808\text{Nmm}.$$

The efficiency η can therefore be determined.

$$\eta = \frac{T_0}{T} = \frac{101808}{661000} = 0.154.$$

■

§12 November 7, 2017

§12.1 Shafts

A shaft is a rotating machine element that either reduces power, transmits power, or both. Shafts are of circular cross-section, and may be solid or hollow depending on the application. The two types of shafts that we will consider are transmission shafts, and machine shafts.

1. Transmission Shafts: These shafts receive power from one end and deliver power at the other end. For example, a motor attached to a pulley at one end of the shaft transmits power to the other end of a shaft with a gear.

2. Machine Shafts: These shafts are an integral part of a machine, or are the main member of a machine. For example, a crank-shaft in an engine.

The material of most shafts are either mostly steel, or medium carbon steel. Generally, materials used for shafts have high strength, good machinability, and low notch sensitivity factors. They also have good heat treatment properties and high wear resistant properties.

The power of shafts is given in killowatts, or horse power.

$$P_{kW} = \frac{Fv}{1000} = \frac{T\omega}{1000} = \frac{T(2\pi n)}{60000} = \frac{Tn}{9549},$$

where F is the force in N, v is the velocity in m/s, T is the torque in Nm, n is the shaft speed in rpm, and ω is the angular velocity in rad/s. In English and British units, we obtain a different expression for power of a shaft.

$$P_{hp} = \frac{Fv}{33000} = \frac{T(2\pi n)}{33000} = \frac{Tn}{5252},$$

where F is the force in lb, v is the velocity in fpm, T is the torque in lbft, and n is the shaft speed in rpm.

§12.2 Design of Shafts

Shafts may be designed on the basis of strength, or rigidity and stiffness. When designing shafts based on strength, we may consider the cases where the shaft is subject to twisting moment or torque only, to bending moment only, to combined twisting and bending moments, or to axial loads in addition to combined torsional and bending loads.

Example 12.1 (Strength Criteria)

A solid shaft transmits 1MW at 240rpm. Determine the diameter of the shaft if the maximum permissible torque exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60MPa.

Solution. We know that we have a solid shaft, the power is 10^6W or 10^3kW, and the speed is 240rpm. We can rearrange the power equation to solve for the mean torque. Doing so, we find that $T = 60(P)/(2\pi n) = 60(10^6)/2(\pi)(240) = 39788.7$Nm. Since T_{max} is twenty percent more, it is $T_{max} = 1.2T = 1.2(39788.7) = 47746$Nm. Since the maximum allowable shear stress is 60MPa, we solve for the strength criteria using the equation

$$\tau = \frac{T_{max}c}{J},$$

where $J = \pi c^4/2$ and c is the radius. Substituting known values, we obtain $60000000 = 2(47746)/\pi c^3$. Solving this gives $c = 0.0797\text{m}$, so $d = 2c = 0.159\text{m}$. ■

Example 12.2 (Rigidity Criteria)

A solid steel shaft transmits 20kW of power at 500rpm. If the maximum torque exceeds the mean torque by 30%, calculate the diameter of the shaft. The shaft is subject to a twist of 1° along a length of 2m. Take $G = 0.84 \cdot 10^5\text{N/mm}^2$.

Solution. We first find the torque using the same strategy as in the previous example. Doing so, we find that $T = 381.97\text{Nm}$. Thus, the maximum torque is $T_{max} = 1.3T = 1.3(381.97) = 496.56\text{Nm}$, which is also 496560Nmm. Using the rigidity criteria, we recall that

$$\phi = \frac{TL}{JG}.$$

Substituting $\phi = \pi/180\text{rad}$, $L = 2000\text{mm}$, and the other known values, we find that the radius is $c = 25.6\text{mm}$, so the diameter is $d = 2c = 51.25\text{mm}$. ■

Example 12.3

A shaft made of mild steel is required to transmit 100kW at 300rpm. The supported length of the shaft is 3m. It carries two pulleys each weighing 1500N supported at a distance of 1m from the ends. Assuming the safe value of shear stress to be 60MPa, determine the diameter of the shaft.

Solution. Since this shaft is subject to both twisting and bending, we will design based on the twisting and bending moment. From the power expression, we find that the torque is 3183000Nmm. Since the beam is symmetrically loaded, we find that the reactions at both ends are 1500N directed upwards. The bending moment at the first pulley (which is the same as the bending moment at the second pulley, and is also the maximum bending moment) is 1500Nm, or 1500000Nmm. The equivalent twisting moment is given by

$$T_{eq} = \sqrt{T^2 + M^2} = \sqrt{3183000^2 + 1500000^2} = 3519000\text{Nmm}.$$

Using $\tau = Tc/J$, we find that the radius $c = 33.4\text{mm}$, so $d = 2c = 2(33.4) = 66.8\text{mm}$. ■

Example 12.4

A line shaft is driven by means of a motor placed vertically below it. The pulley on the line shaft is 1.5m in diameter and has belt tensions of 5.4kN and 1.8kN on the tight side and on the slack side of the belt respectively. Both of these tensions may be assumed to be vertical. If the pulley is overhung from the shaft, with the distance of the centerline of the pulley from the centerline of the bearing being 400mm, determine the diameter of the shaft. Assume the maximum allowable shear stress to be 42MPa.

Solution. Note that the torque transmitted by the pulley is given by

$$T = (T_1 - T_2)R,$$

where T_1 is the larger force of 5.4kN, T_2 is the smaller force of 1.8kN, and R is half of the diameter of the pulley. Substituting values, we find that $T = 2700$Nm. The total load due to the belt tension is

$$W = T_1 + T_2 = 5400 + 1800 = 7200\text{N}.$$

Thus, the bending moment acting on the shaft is $M = 7200(400) = 2880$Nm. The equivalent twisting moment is therefore

$$T_{eq} = \sqrt{T^2 + M^2} = \sqrt{2700^2 + 2880^2} = 3950000\text{Nmm}.$$

Now, equating this with the strength criteria for the maximum allowable shear stress of 42MPa using the formula $\tau = Tc/J$, we find that the radius is $c = 39.12$mm, so $d = 2c = 2(39.12) = 78$mm. ■

§13 November 9, 2017

§13.1 Shaft Example

Example 13.1

Suppose we have a shaft with ends A and D, where A is to the left of B by 200mm, B is left of C by 600mm, and C is left of D by 200mm. B and C are pulleys. Tension T_1 and T_2 act downwards on both sides of pulley B, and T_3 and T_4 act on both sides of C to the right, when viewed from A to D. The diameter of C is 250mm, and the diameter of B is 500mm. Power is supplied by means of the vertical belt on pulley B that is transmitted to pulley C carrying the horizontal belt. The maximum tension of pulley B is 2.5kN, the angle of the wrap of the belts on the pulleys is $\theta = 180°$, the coefficient of friction is $\mu = 0.24$, the factor of safety is $FS = 3$, and the maximum stress is $S_y = 400$MPa.

Solution. To solve these kinds of problems, we make repeated use of the following properties.

$$\frac{T_1}{T_2} = e^{\mu\theta},$$
$$T = (T_1 - T_2)R,$$

where T_1 is the greater tension compared to T_2, μ is the coefficient of friction, θ is the angle around the pulley for which the tensions act, and R is the radius of the pulley.

We know that the maximum tension on pulley B is $T_1 = 2500$N, so we want to apply the first equation above to determine the tension on the other side of the pulley. With $\mu = 0.24$ and $\theta = 180° = \pi$, we find that

$$T_2 = \frac{T_1}{e^{\mu\pi}} = \frac{2500}{e^{0.24\pi}} = 1176\text{N}.$$

Using the second equation above, we find

$$T = (T_1 - T_2)R_1 = (2500 - 1176)250 = 330882\text{Nmm}.$$

We still need to find T_3 and T_4 at C. But $(T_1 - T_2)R_1 = (T_3 - T_4)R_2$ since the torque is the same on the shaft. We also know that $T_3/T_4 = e^{\mu\theta}$, so we can simultaneously solve these two equations. Doing so, we find $T_3 = 5000$N and $T_4 = 2353$N.

Now, we draw bending moment diagrams in both the vertical and horizontal direction. First drawing the diagram in the vertical direction, we have $T_1 + T_2 = 3676$N downwards at B. We can find the reaction forces at A and B in the vertical plane. Doing so, we find $A_y = 2941$N and $B_y = 735$N. From the bending moment diagram, we find that $M_B = 588200$Nmm and $M_C = 147062$Nmm. Now, we apply the above reasoning in the horizontal direction. $T_3 + T_4 = 7353$N and the corresponding reaction forces are $A_x = 1470.6$N and $B_x = 5882.4$N. From the bending moment diagram, this gives $M_B = 294120$Nmm and $M_C = 1176480$Nmm.

We now find the resultant moments at B and C by combining the horizontal and vertical terms. For B, this is

$$M_B = \sqrt{M_{Bx}^2 + M_{By}^2} = \sqrt{294120^2 + 588200^2} = 657636.5\text{Nmm}.$$

For C, we obtain $M_C = 1185635.9$Nmm.

Thus, we can determine the equivalent twisting moment by considering the torque $T = 330882$Nmm with the greater of the two moments found previously.

$$T_{eq} = \sqrt{M^2 + T^2} = \sqrt{1185625.45^2 + 330882^2} = 1230940.85\text{Nmm}.$$

Now, we apply the equation $\tau = T_{eq}c/J$. But recall that the maximum allowable shear stress is $\tau_{max} = (S_y/2)/FS = 400/6 = 66.7$MPa. Thus, using this value of τ, we obtain the radius $c = 22.7$mm, so $d = 2c = 2(22.7) = 45$mm. ■

§13.2 Designing Shafts for Stress

For moment, we have the amplitude of oscillation M_a that goes from the amplitude to the peak, and the offset of oscillation M_m which goes from the bottom to the average.

$$\sigma_m = K_f \frac{32M_m}{\pi d^3},$$

$$\sigma_a = K_f \frac{32M_a}{\pi d^3},$$

where K_f is the fatigue stress concentration factor that can be found from $K_f = 1 + (K_t - 1)q$. For torque, we similarly have the amplitude of oscillation T_a that goes from the amplitude to the peak, and the offset of oscillation T_m which goes from the bottom to the average.

$$\tau_m = K_f \frac{16T_m}{\pi d^3},$$

$$\tau_a = K_f \frac{16T_a}{\pi d^3}.$$

Usually, we neglect axial loading when we design a shaft. This is because the important loadings to consider are bending and twisting. For distortion energy failure theory, we utilize fatigue stress concentration factor K_f and fatigue stress concentration factor for shear stress K_{fs}.

$$\overline{\sigma}_a = \sqrt{\sigma_a^2 + 3\tau_a^2} = \sqrt{\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T_a}{\pi d^3}\right)^2},$$

$$\overline{\sigma}_m = \sqrt{\sigma_m^2 + 3\tau_m^2} = \sqrt{\left(\frac{32K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T_m}{\pi d^3}\right)^2}.$$

For each of the following methods, we solve for the diameter d or the factor of safety FS.

- **DE Goodman Method**:

$$\frac{1}{FS} = \frac{16}{\pi d^3}\left(\frac{1}{S_n}\sqrt{4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2} + \frac{1}{S_u}\sqrt{4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2}\right),$$

$$d = \left(\frac{16n}{\pi}\left(\frac{1}{S_n}\sqrt{4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2} + \frac{1}{S_u}\sqrt{4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2}\right)\right)^{1/3}.$$

- **DE ASME Elliptic Method**:

$$\frac{1}{FS} = \frac{16}{\pi d^3}\sqrt{4\left(\frac{K_f M_a}{S_n}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2},$$

$$d = \left(\frac{16n}{\pi}\sqrt{4\left(\frac{K_f M_a}{S_n}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2}\right)^{1/3}.$$

- **DE Soderberg Method**:

$$\frac{1}{FS} = \frac{16}{\pi d^3}\left(\frac{1}{S_n}\sqrt{4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2} + \frac{1}{S_y}\sqrt{4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2}\right),$$

$$d = \left(\frac{16n}{\pi}\left(\frac{1}{S_n}\sqrt{4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2} + \frac{1}{S_y}\sqrt{4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2}\right)\right)^{1/3}.$$

- **DE Gerber Method**:

$$\frac{1}{FS} = \frac{8A}{\pi d^3 S_n}\left(1 + \sqrt{1 + \left(\frac{2BS_n}{AS_u}\right)^2}\right),$$

$$d = \left(\frac{8nA}{\pi S_n}\left(1 + \sqrt{1 + \left(\frac{2BS_n}{AS_u}\right)^2}\right)\right)^{1/3},$$

where A and B are defined as

$$A = \sqrt{4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2},$$
$$B = \sqrt{4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2}.$$

In the case of a stationary shaft, we have $M_m = 0$, so the offset is zero, and we just have the amplitude of oscillation. We have a stationary (or static) shaft that is rotating (completely reversed). When $T_a = 0$, this means that there is no amplitude of oscillation, so the shaft is steady. Static failure occurs at the failure point

§14 November 16, 2017

§15 November 21, 2017

§15.1 Keys

A *key* is a piece of metal which is used to connect a shaft and a hub (boss) or hollow shaft in order to prevent relative motion. The key material is the same as the shaft material, and is subject to shear and crushing stresses. The following are different types of keys.

- **Sunk Keys**: Half is inside the shaft, and the other half is inside the hub. Sunk keys include rectangular keys, square sunk keys, parallel sunk keys, gib-head keys, feather keys, and woodruff keys. For rectangular and square keys, refer to a table by looking up the values required at a given shaft diameter. All that remains is to find the length of the key. Gib head keys look similar to square keys with a head at one end, and are used for easy assembly or disassembly. Based on the shaft diameter, we can find the width w and thickness t from Table 13.1 of Machine Design by R. S. Khurmi and J. K. Gupta.

- **Saddle Keys**: This includes both flat saddle and hollow saddle keys. A flat saddle key is a taper key that fits in the keyway in the hub and is flat on the shaft. It is likely to slip around the shaft under load, so it is used for relatively light loads. A hollow saddle key is a taper key that is shaped to fit the curved surface of the shaft. They are suitable for temporary fastening.

- **Tangent Keys**: These are fitted in pairs at right angles. Each key is to withstand torsion in one direction only. These types of keys are generally used in heavy duty shafts.

- **Round Keys**: They are circular in section and fit into holes drilled between the shaft and the hub. They are advantageous in that their keyways may be drilled after the mating parts are assembled. Round keys are most appropriate for lower power drives.

- **Splines**: These keys are made integrated into the shaft which fits in the keyway broached in the hub. These splined shafts have four, six, ten, or even sixteens splines. Splined shafts are usually used when the force transmitted is large compared to the size of the shaft. For instance, they are used in automobile transmission and sliding gear transmissions.

§15.2 Strength of Keys

We may consider various types of failure for the key. When solving problems, we generally solve for length, and take the length to be the longer of the two obtained when considering shear and crushing stresses.

1. Considering the shear failure of the key, we note that the area resisting the shear is given by $A = wl$, where w is the width, and l is the length. The tangential force is given by $F_t = wl\tau$. Since the torque transmitted by the key is given by $T = F_t d/2$. Thus, considering shear stress only, we find that

$$T = \frac{wl\tau d}{2},$$

   where d is the shaft diameter, τ is the shear stress, w is the width of the key obtained from the table, l is the length of the key, and T is the torque obtained.

2. Considering the crushing are of $tl/2$, we find that $F_t = tl\sigma_{cr}/2$. Thus, the torque due to the crushing force is

$$T = \frac{tl\sigma_{cr}d}{4},$$

   where t is the thickness, and σ_{cr} is the crushing stress.

To find the length of the key that permits transmission of full power of the shaft, we equate the shearing strength of the key to the torsional shear strength of the shaft.

Example 15.1

Design the rectangular key for a shaft of 50mm diameter. The shearing and crushing stress for the key material are 42MPa and 70MPa respectively.

Solution. Referring to the table, we find that $w = 16$mm, and $t = 10$mm for a shaft diameter of 50mm. Additionally, we are given that $\tau = 42$MPa and $\sigma_{cr} = 70$MPa. Therefore, considering the shearing of the key, we find that the torque is $T = wl\tau d/2 = 16(l)(42)(50)/2 = 16800l$Nmm. But from the standard equation for torque, we have $T = \pi\tau d^3/16 = \pi(42)(50)^3/16 = 1.03 \cdot 10^6$Nmm. Thus, solving for l, we find that $l = 61.31$mm according to shearing stress.

Considering the crushing of the key, we find that $T = tl\sigma_{cr}d/4 = (1)(l)(70)(50)/4 = 8750l$Nmm. From the previous calculation for torque where it was found that $T = 1.03 \cdot 10^6$Nmm, we find that $l = 117.7$mm. We use the length for the length of the rectangular key. ■

Example 15.2

A 45mm diameter shaft is made of steel with a yield strength of 400MPa. A parallel key of size 14mm wide and 9mm thick made of steel with a yield strength of 340MPa is to be used. Find the required length of the key if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of safety equal to 2.

Solution. From the maximum shear stress theory, we have $\tau_{max} = Sy/4 = 400/4 = 100$MPa, since the factor of safety is 2. Now, we can use this to find the torque from $T = \pi\tau d^3/16 = \pi(100)(45)^3/16 = 1.8 \cdot 10^6$Nmm. Applying the same for the key, we find $\tau = 340/4 = 85$MPa, and $T = wl\tau d/2 = (14)(l)(85)(45)/2$. Solving with $T = 1.8 \cdot 10^6$Nmm from before, we find that $l = 67.2$mm.

Now with regards to crushing stress, we find that $\sigma_{cr} = S_y/FOS = 340/2 = 170$Nmm. But $T = tl\sigma_{cr}d/4 = (9)(l)((170)(45)/4 = 17213l$. Solving with $T = 1.8 \cdot 10^6$Nmm, we find that $l = 104.6$mm. Thus, we choose the longer length as the length of the key. ■

§15.3 Coupling

Since long shafts are inconvenient to transport, shorter shafts are often connected by means of a *coupling*. A coupling is a mechanical element which connects two shafts together. It is used for connection, alignment, and shock loading. Flange coupling applies to coupling with two separate cast iron flanges. Each flange is mounted on the shaft end and keyed to it. The design procedure is described below.

1. Design for the Hub: The hub is designed by considering it as a hollow shaft, transmitting the same torque T as that of a solid shaft.

$$T = \frac{\pi\tau_c}{16}\left(\frac{D^4 - d^4}{D}\right),$$

where τ_c is the allowable shear stress of the flange (cast iron), D is the outer diameter of the hub, and d is the diameter of the shaft. The outer diameter of hub is usually taken as $2d$, and the length of the hub as $1.5d$. The induced shearing stress in the hub may be checked using the torque equation above.

2. Design for Key: The key is designed with usual proportions and then checked for shearing and crushing stresses. The material of the key is generally the same as the shaft, and the length of the key is equal to the length of the hub, L. Use the key design procedure described previously.

3. Design for Flange: The flange is under shear while transmitting the torque.

$$T = \frac{\pi D^2 \tau_c t_f}{2},$$

where the thickness of flange is usually taken as $t_f = 0.5d$. Therefore, the induced shearing stress in the flange may be checked agains the torque equation above. The outer diameter of the flange is taken to be $D_3 = 4d$, while the thickness of the protective flange is taken to be $t_p = 0.25d$.

4. Design for Bolts: The bolts are subjected to shear stress due to the torque transmitted. The number of bolts n depends upon the diameter of shaft, and the pitch circle diameter of the bolts $D_1 = 3d$.

$$T = \frac{\pi (d_1)^2 \tau_b n}{4} \left(\frac{D_1}{2}\right),$$

where τ_b is the allowable shear stress for the bold, and the diameter of bolt d_1 may be obtained by substituting known values. After obtaining the diameter of the bolt d_1, we can optionally check against the crushing stress in the bolts,

$$T = \frac{n d_1 t_f \sigma_{cb} D_1}{2},$$

where σ_{cb} is the allowable crushing stress for the bolt.

Example 15.3

Design a cast iron flange coupling for a mild steel shaft transmitting 90kW at 250rpm. The allowable shear in the shaft is 40MPa. The angle of twist is not to exceed $\phi = \pi/180$rad in a length of $20d$, where d is the diameter. The allowable shear stress for coupling bolts is 30MPa, and the modulus of rigidity is $G = 84\text{kN/mm}^2$. The cast iron shear stress is 14MPa.

Solution. We note that the power is 90000W, and the speed is $n = 250$rpm. We can find the torque from $P = 2\pi nT/60$. Solving, we find that $T = 90000(60)/(2\pi(250)) = 3440$Nm. Setting this equal to $T = \pi\tau d^3/16 = \pi(40)(d)^3/16$, we find that $d = 76$mm. This is based on the **strength criteria**. Now, we solve according to the **rigidity criteria**. From here, we find the diameter d from $\phi = TL/(JG)$ by using $T = 3440$Nm and the values given. We find that $d = 78$mm in this case. Thus, choosing the larger of the two, we choose $d = 78$mm, so $l = 156$mm.

Now, we design the hub by approximating the shaft diameter as 80mm, where $D_{hub} = 2D_{shaft}$. Thus, $D_{hub} = 2(80) = 160$mm. The length of the hub is $L_{hub} = 1.5D_{shaft}$, so this means that $L_{hub} = 1.5(80) = 120$mm. We will consider the hub as a hollow shaft subject to shear. Assuming that the flange and the hub are made of the same material (cast iron), then $\tau_{allow} = 14$MPA. Therefore, the torque transmitted by the hub is $T = \pi\tau_{hub}\left(D_1^4 - D^4\right)/16D_1 = 3440 \cdot 10^3 = \pi\tau_{hub}\left(160^4 - 80^4\right)/16(160)$. This results in $\tau_{hub} = 4.56\text{N/mm}^2$. Since the shear stress of the hub is less than the allowable shear stress, then the hub is safe under shear.

Next, we will design the key. First, we must select the key dimensions for the 80mm diameter of the shaft. Referring to the table, we obtain the width and the thickness. For 80mm, we find that $w = 25$mm and $t = 14$mm. Considering the key under shear stress, $T = wl\tau D/2$. Substituting, we find that $3440 \cdot 10^3$Nmm $= 25(120)\tau(80)/2$. We find that $\tau = 28.7$N/mm^2, which is less than the allowable shear in the shaft of $40MPa$. Note that we do not consider crushing stress, since the problem does not give information on this.

We will design the flange. Let $D_3 = 4D_{shaft} = 4(80) = 320$mm be the outer diameter of the flange. The thickness of the flange is $t_f = 0.5D = 0.5(80) = 40$mm. The thickness of the protective flange is $t_p = 0.25D_{shaft} = 0.25(80) = 20$mm. We want to see whether this is safe under shear. Considering the shear failure of the flange, the shearing area is $\pi D_1 t_f$. The shearing strength of the flange gives $T = \pi D_1 t_f \tau D_1/2 = 3400000 = \pi(160)(40)\tau(160)/2$. Solving, we find $\tau = 2.14$N/mm^2. Since this value is less than the permissible shear of 14MPa, the flange is find under shear.

Finally, we design the bolts. Let $D_2 = 3D_{shaft} = 3(80) = 240$mm be the pitch circle diameter for the bolts. Considering the bolts under shear, the shearing area is $A = \pi d_c^2/4$. Thus, letting $n = 4$ be the number of bolts, we can determine the diameter d_c. $T = An\tau_{bolt}D_2/2 = 340000 = \pi d_c^2(4)(30)(240)/8$. Solving, we find $d_c^2 = 304$, so $d_c = 17.4$mm. Rounding up, we obtain $d_c = 20$mm.

■

§16 November 23, 2017

§16.1 Design of Rolling-Element Bearings

The materials for bearings. Ball bearings including rings and balls are usually made from high carbon chrome steel. Roller bearing elements are made from carburized alloy steel. The cleanliness of the steel is extremely important, and tolerance is critical as well. When the load angle $\alpha = 0°$, we have a *radial ball bearing*, while $\alpha = 25°$ means we have an angular ball bearing.

When calculating the life of a known bearing, we first refer to Table 14.1 first to obtain the bore diameter of the specified bearing. Then, we refer to Table 14.2 to obtain the rated capacity C corresponding to that series and bore diameter. When determining which bearing to use, we arrive at a rated capacity using the formulas introduced below. Then, we choose the next largest C in each of the relevant bearing series from Table 14.2 in order to find the associated bore diameter. This allows us to use Table 14.1 to determine each of the permissible bearings that could be used for this application.

When selecting bearings, we are often concerned with the life, and the reliability of bearings. Designing for life, we use the exponent $e = 10/3$ for all bearings. The life L of the bearing is given as follows.

$$L = L_R\left(\frac{C}{F_r}\right)^{3.33},$$

$$C = F_r\left(\frac{L}{L_R}\right)^{0.3},$$

where C is the rated capacity found in Table 14.2, L_R is the life corresponding to the rated capacity ($9 \cdot 10^6$ revolutions), F_r is the radial load, and L is the life corresponding to the radial load.

When we also want to factor in reliability as well, we include the reliability factor K_r.

$$L = K_r L_R\left(\frac{C}{F_r}\right)^{3.33},$$

$$C = F_r \left(\frac{L}{K_r L_R} \right)^{0.3},$$

where the reliability factor K_r is dependent on the percentage of reliability desired from Figure 14.3.

Occasionally, we may also be interested in supported an axial load with the bearings. For ball bearings, any combination of radial load F_r and thrust load F_t result in approximately the same life as a pure radial equivalent load F_e which is calculated as follows. For $\alpha = 0$,

$$F_e = \begin{cases} F_r & \text{if } 0 < F_t/F_r < 0.35, \\ F_r \left(1 + 1.115 \left(\frac{F_t}{F_r} - 0.35\right)\right) & \text{if } 0.35 < F_t/F_r < 10, \\ 1.176 F_t & \text{if } F_t/F_r > 10. \end{cases}$$

For $\alpha = 25$,

$$F_e = \begin{cases} F_r & \text{if } 0 < F_t/F_r < 0.68, \\ F_r \left(1 + 0.870 \left(\frac{F_t}{F_r} - 0.68\right)\right) & \text{if } 0.68 < F_t/F_r < 10, \\ 0.991 F_t & \text{if } F_t/F_r > 10. \end{cases}$$

Because the standard rated capacities are for conditions of uniform load without shock, we may also need to introduce an application factor K_a when shock loading occurs. This can be obtained from the following table.

Type of Application s	Ball Bearing	Roller Bearing
Uniform load, no impact	1.0	1.0
Gearing	1.0-1.3	1.0
Light Impact	1.2-1.5	1.0-1.1
Moderate Impact	1.5-2.0	1.1-1.5
Heavy Impact	2.0-3.0	1.5-2.0

Now, we can substitute F_e the equivalent load for F_r and introduce K_a to obtain the following equations.

$$L = K_r L_r \left(\frac{C}{F_e K_a} \right)^{3.33},$$

$$C = F_e K_a \left(\frac{L}{K_r L_R} \right)^{0.3},$$

where the life can be chosen from the following table.

Type of Application	Design Life (thousands of hours)
Instruments and apparatus for infrequent use	0.1-0.5
Machines used intermittently, where service interruption is of minor importance	4-8
Machines intermittently used, where reliability is of great importance	8-14
Machines for 8-hour service, but not every day	14-20
Machines for 8-hour service, every working day	20-30
Machines for continuous 24-hour service	50-60
Machines for continuous 24-hour service where reliability is of extreme importance	100-200

> **Example 16.1**
> Select a ball bearing for an industrial machine press fit onto a shaft and intended for continuous one shift (8 hours/day) operation at 1800 rpm. The radial and thrust loads are 1.2kN and 1.5kN respectively. Light to moderate load impact .

Solution. We know that $F_t = 1.5\text{kN}$ and $F_r = 1.3\text{kN}$. Since we have light to moderate impact for the ball bearing, we find from Table 14.3 that $K_a = 1.5$. Additionally, $n = 1800\text{rpm}$ and they are used for 8h/day. $F_t/F_r = 1.5/1.2 = 1.25$. When $\alpha = 0$, we find that the ratio F_t/F_r is between 0.35 and 10, so for radial bearing,

$$F_e = F_r\,(1 + 1.115(F_t/F_r - 0.35)) = 1.2\,(1 + 1.115(1.25 - 0.35)) = 2.4\text{kN}.$$

For $\alpha = 25°$, we have angular bearing,

$$F_e = F_r\left(1 + 0.870\left(\frac{F_t}{F_r} - 0.68\right)\right) = 1.2(1 + 0.870(1.25 - 0.68)) = 1.8\text{kN}.$$

Since we know K_a, and we know that the life $L = 30000 \cdot 1800 \cdot 60\text{min/h} = 3240 \cdot 10^6$ revolutions from Table 14.4 (working 8 hour service every day, we choose 30 thousand), we can combine this with 90% reliability from Table 14.13 to obtain $K_r = 1$. Therefore, we find for the radial bearing that

$$C = 2.4(1.5)\left(\frac{3240 \cdot 10^6}{90 \cdot 10^6}\right)^{0.3} = 10.55\text{kN},$$

For the angular result, we find

$$C = 1.8(1.5)\left(\frac{3240}{90}\right)^{0.3} = 7.9\text{kN}.$$

Now, from Table 14.2, we choose the next largest C of any of the possible radial/angular bearing choices. These have a corresponding bore diameter. Then, refer to Table 14.3 to obtain the exact bearing type of the series by matching with the bore diameter. ■

§17 December 5, 2017

§17.1 Spur Gears

Gears are toothed members that transmit rotary motion from one shaft to another. *Spur gears* in particular are the most common type of gears, and are used to transfer motion between parallel shafts using teeth that are parallel to the shaft axes.

In any pair of mating gears, the smaller of the two is called the *pinion* while the larger is called the *gear*. Using the subscripts p and g to denote the pinion and gear respectively, we can relate the pitch diameters d with the angular velocities ω, and define the center distance c as follows.

$$\frac{\omega_p}{\omega_g} = -\frac{d_g}{d_p},$$
$$c = \frac{d_p + d_g}{2} = r_p + r_g.$$

The circular pitch p is defined along the pitch circle by the distance between the start of each each tooth. However, more commonly used indices of gear-tooth size are diametral pitch P (used with English units only), and module m (used with metric units only).

$$p = \frac{\pi d}{N},$$

$$P = \frac{N}{d},$$

$$m = \frac{d}{N},$$

where d is the pitch diameter, N is the number of teeth in the gear or pinion, P is measured in teeth per inch, and m is in millimeters per teeth. Note that the most commonly used pressure angle with both English and metric units is $\phi = 20°$. In the United States, 25° is also standard. The face width b must be within the ranges $9/P < b < 14/P$ or $9m < b < 14m$.

The maximum possible addendum circle for the gear or pinion is given by the following formula.

$$r_{a,max} = \sqrt{r_b^2 + c^2 \sin^2(\phi)},$$

where r_b is the base circle radius of the pinion or gear, and ϕ is the pressure angle. The average number of teeth in contact as the gears rotate together is known as the contact ratio CR.

$$CR = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - c\sin(\phi)}{p_b},$$

where r_{ap} and r_{ag} are the addendum radius of the pinion and gear, and r_{bp} and r_{bg} are the base circle radius of the pinion and gear. We can now define the base pitch p_b, and the radius/diameter of the base circle.

$$p_b = \frac{\pi d_b}{N} = p\cos(\phi),$$

$$d_b = d\cos(\phi),$$

$$r_b = r\cos(\phi).$$

In general, the greater the contact ratio, the smoother and quieter the operation of the gears. A contact ratio of 2 or more means that at least two pairs of teeth are theoretically in contact at all times.

Example 17.1

Two parallel shafts with 4in center distance are to be connected by 6-pitch, 20° spur gears providing a velocity ratio of -3.0. Determine the pitch diameters and numbers of teeth in the pinion and gear. Determine whether there will be interference when standard full-depth teeth are used. Determine the contact ratio.

Solution. $c = r_p + r_g$, where $c = 4$in. We also have $r_g/r_p = -\omega_p/\omega_g = -(-3) = 3$. Solving both of these, we find that $r_p = 1$ and $r_g = 3$. Thus, $d_p = 2$in and $d_g = 6$in. Knowing that the diametral pitch $P = 6$, we have $P = N/d$, so $N_p = 2(6) = 12$ teeth and $N_g = 6(6) = 36$ teeth. We have $r_{bp} = r_b\cos(\phi) = 1\cos(20)$ and $r_{bg} = r_g\cos(\phi) = 3\cos(2)$. Thus, we apply the formula for $r_{a,max}$ to find $r_{ap,max} = 1.66$in and $r_{ag,max} = 3.133$in. The limiting outer gear radius results in an addendum of 0.133in, which is less than the standard full-depth tooth addendum of $1/P = 1/6 = 0.167$in. Thus, the use of standard teeth would result in interference.

Thus, we will choose unequal addenda gears with $a_g = 0.060$in and $a_p = 0.290$in. These are chosen to provide maximum addenda for greatest contact ratio, while limiting the gear addendum to prevent interference, and limiting the pinion addendum to maintain the width of the top land. Substitution into equations results in $p_b = (\pi/6)\cos(20) = 0.492$in, $r_{ap} = 1.290$in, $r_{bp} = 1\cos(20)$, $r_{ag} = 3.060$in, $r_{bg} = 3\cos(20)$. This gives a contact ratio of $CR = 1.43$ after substituting values, which is acceptable. ■

The force between mating teeth can be resolved at the pitch point into two components. The tangential component F_t when multiplied by the pitch line velocity V accounts for the power transmitted, while the radial component F_r does no work but tends to push gears apart.

$$F_r = F_t \tan(\phi).$$

To analyze the relationships between gear force components and the associated shaft power and rotating speed, we use the following formulas in English units.

$$V = \frac{\pi d n}{12},$$

$$P_{hp} = \frac{F_t V}{33000},$$

where d is the pitch diameter in inches, n is the revolutions per minute, V is in feet per minute, F_t is in pounds, and P_{hp} is the power in horsepower. For metric units, we have

$$V = \frac{\pi d n}{60000},$$

$$P_W = F_t V,$$

where d is in millimeters, n is in rpm, V is in meters per second, F_t is in Newtons, and P_W is the power in Watts.

Example 17.2

Let the diametral pitch be $P = 3$, $\phi = 20$, $n = 600$rpm, and $P_{hp} = 25$hp. Determine F_t and F_r.

Solution. First, we apply equation find that $d_{pitch} = N_a/P$, where N_a is the number of teeth and $P = 3$ is the diametral pitch. Thus with 12 teeth, we have $d = 4$in. Next, we find the pitch line velocity $V = \pi dn/12 = \pi(4)(600)/12 = 628.28$ft/min. We now apply $P_{hp} = F_t V/33000$ and solve for $F_t = 33000(25)/628.28 = 1313$lb. Now, $F_r = F_t \tan(20) = 1313 \tan(20) = 478$lb. Thus, the total gear tooth forces are $F_t + F_r = 1791$lb acting both vertically and horizontally, resulting in a vector sum of $1719\sqrt{2} = 2533$lb acting at $45°$ as the resultant load applied by the idler to its shaft. ∎

§18 Quiz

Bearing chapter 14 from book[On FINALexam worth around 5%] spur gear chapter 15 coupling chapter 13 from other book.

rigid coupling , flange, one secured to each shaft. sleeve used for long shafts, or those that can be aligned well.

flexible coupling take into account misalignment. chain coupling gear coupling, gear mounted on shaft,

§18.1 Final

Shaft design lecture and tutorial coupling designe (protected flange, hush pin) textbook no notes. (Chapter 17, shaft, coupling, keys) rolling bearing lement, spur gear chapter 13 tetbook of machine design khurmi,

§18.2 Final, December 5

power screw chapter 10, lecture and tutorial coupling (hub, key, shaft, bolt, flange) chapter 13 bearing chapter 14, lecture and tutorial shaft chapter 17, lecture and tutorial (SELECTED PROBLEMS) spur gear, tutorial 8.